



Course Title: Engineering Mathematics (3) b Second Year (Computer & Automatic control Department)  
Course Code: PME2111 Date: 9 / 6 / 2013 (Second term) Allowed time: 3 hrs No. of Pages: (2)

**Remarks:** (Answer the following questions. Assume any missing data...)

### Problem number 1(15 Marks)

- (a) Find an analytic function whose real part is  $u(x, y) = \sin x \cosh y$ .  
 (b) Show that if  $f(z) = u(r, \theta) + iv(r, \theta)$  is analytic, then  $r^2 u_{rr} + r u_r + u_{\theta\theta} = 0$ .  
 (c) (Cauchy's Theorem) Prove that If  $f(z)$  is analytic in a simply-connected region  $D$ , then for every simple closed curve  $C$  in  $D$ ,  $\oint_C f(z) dz = 0$

### Problem number 2(15 Marks)

(a) Evaluate

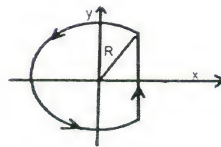
(i)  $\oint_C \frac{z^3 + 1}{(z-1)(z-5)} dz$ ,  $C: |z| = 3$ . (ii)  $\oint_C \frac{\sinh 3z}{(z-1)^4} dz$ ,  $C: |z-1| = 3$ .

(iii)  $\oint_{|z|=2} z^2 \cosh \frac{2}{z-1} dz$

(b) Find Laurent's expansion of  $f(z) = \frac{1}{z^2 - 3z + 2}$  on the regions

(i)  $1 < |z| < 2$  (ii)  $0 < |z-1| < 1$

(c) Using Bromwich contour



To find inverse Laplace transform of  $F(s) = \frac{\cosh x \sqrt{s}}{s(s^2 + 1)}$ ,  $0 < x < 1$

### Problem number 3(15 Marks)

(a) Find  $\sup(A)$ , center of  $A$ , height of  $A$  and relative cardinality of  $(\|A\|)$

where  $A$  is fuzzy set  $A = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.2}{c} + \frac{0.8}{d}$

(b) Show that the set  $A = \int \frac{1}{(1+x^2)^2}$  is convex

(c) Show that any membership  $\mu_{A \cup B}$  satisfy S-norm axiom satisfy

$$\max(\mu_A(x), \mu_B(x)) \leq \mu_{A \cup B} \leq \mu_{ds}(x)$$

where  $\mu_{ds}(x) = \begin{cases} \mu_A(x) & \text{if } \mu_B(x) = 0 \\ \mu_B(x) & \text{if } \mu_A(x) = 0 \\ 1 & \text{otherwise} \end{cases}$

#### **Problem number 4 (15 Marks)**

(a) Show that Sugeno fuzzy complements class satisfies the complement Axioms. (hint:  $C_\lambda(a) = \frac{1-a}{1+\lambda a}$ ,  $0 < \lambda < \infty$  and  $a = \mu_A(x)$ )

(b) Find the fuzzy distance between elements of two fuzzy sets

$$A = \frac{0.1}{1} + \frac{0.3}{2} + \frac{0.7}{3} \quad \text{and} \quad B = \frac{0.5}{1} + \frac{0.2}{3} + \frac{0.6}{5}$$

(c) Let fuzzy set A be the set of people with an infectious disease and the crisp set B be the sets of people having been in contact with the infected people and C be the crisp set of people contact with B. The contact relations is given by R1 and R2

$$R_1 = \frac{0.8}{(a_1, b_1)} + \frac{0.2}{(a_2, b_2)} + \frac{0.3}{(a_3, b_1)} + \frac{0.7}{(a_4, b_2)} + \frac{0.4}{(a_4, b_3)}, R_2 = \frac{0.1}{(b_1, c_1)} + \frac{0.2}{(b_2, c_2)} + \frac{0.5}{(b_3, c_1)} + \frac{0.9}{(b_3, c_3)}$$

$$A = \{(a_1, 0.4), (a_2, 0.5), (a_3, 0.9), (a_4, 0.6)\}, B = \{b_1, b_2, b_3\} \quad \text{and}$$

$$C = \{c_1, c_2, c_3\}. \quad \text{Find the fuzzy sets } \tilde{B} \text{ and } \tilde{C}.$$

#### **Problem number 5 (25Mark)**

(a) Evaluate the following integrals:

$$1) \int_3^\infty e^{6x-x^2} dx \quad 2) \int_2^5 (x-2)\sqrt{5-x} dx \quad 3) \int_0^\infty \frac{dy}{1+y^4}$$

(b) Find the series solution of the D.E.  $(1+x^2)y'' + y = 0$

(c) Prove that: 1)  $\frac{d}{dx} J_0(x) = -J_1(x)$  2)  $\int x^4 J_1(x) dx = x^4 J_2 - 2x^3 J_3 + c$

*End*

*All best wishes*

*Prof. Dr. A. Abo Khadra, Dr .M. Shokry, Dr. Assem Elshenawy and the committee*